

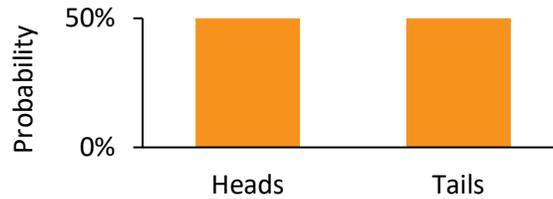
Simulations in Finance

This document shows how to run Monte Carlo simulations using Crystal Ball. It starts with a simple example of coin tosses to cover the intuition and basics of simulations. Next follow two finance applications:

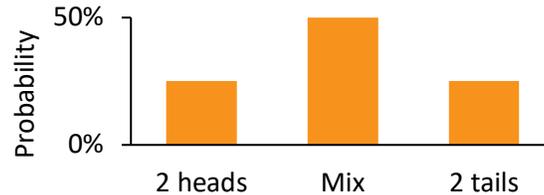
- a portfolio example with three securities, in which the future portfolio value is simulated, and
- a corporate liquidity example, in which the future cash holdings for a firm are simulated.

1. A coin toss example

Let us start with a simple example of coin tosses. If you toss a coin once, the probability of either heads or tails is 50%, with the probability distribution given below.



If you toss the coin twice, the probability of two heads is $0.5 \times 0.5 = 0.25$, the probability of two tails is also $0.5 \times 0.5 = 0.25$, and the probability of one of each is 0.5.



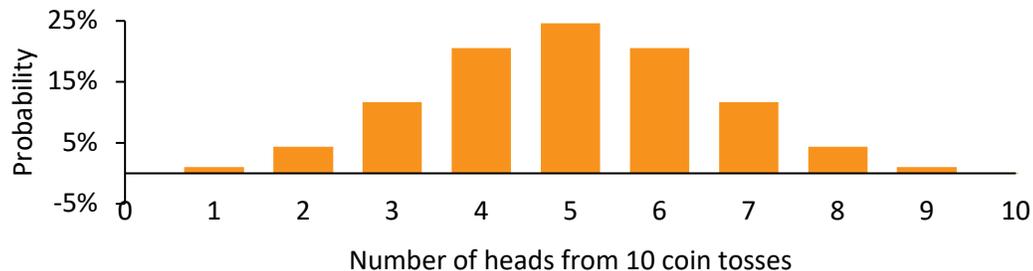
If you toss the coin ten times, we can use the binomial distribution formula to estimate the probabilities:

$$\text{Probability of } k \text{ heads} = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

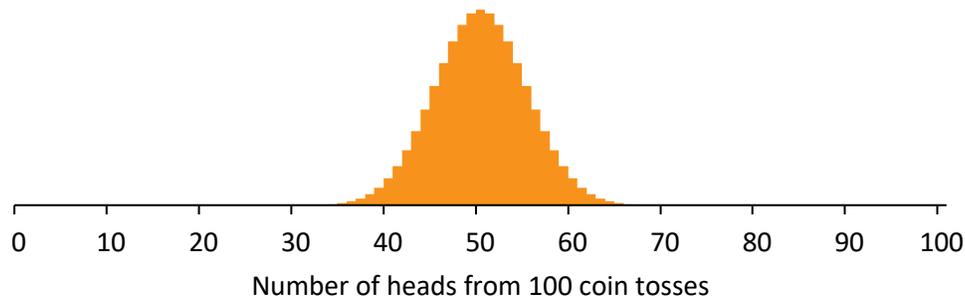
where n is the number of coin tosses and p is the probability of heads in each toss. For example, the probability of five heads is:

$$\text{Probability of 5 heads} = \frac{10!}{5!(10-5)!} 0.5^5 (1-0.5)^{10-5} = 0.246$$

The complete probability distribution is given below.



Lastly, the distribution for 100 coin tosses is given below, and you can see that it is converging toward a normal distribution.



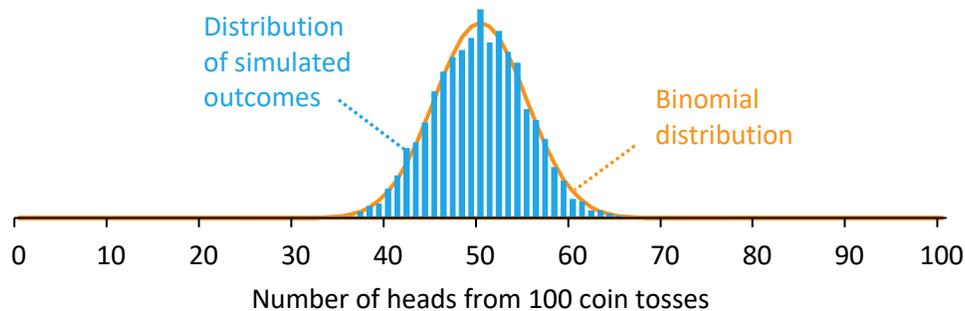
Let us simulate the distribution instead. We could toss 100 coins and record the number of heads, and then repeat this 5,000 times to form an empirical distribution. To save some time, I employed Excel:

- Column A contains the numbers 1 through 100 to label each of the 100 coin tosses.
- Column B contains a random number function, =RAND(), which gives a random value between 0 and 1 using a uniform distribution. We can then think of this as a coin toss where a value above 0.5 means heads.
- Column D contains the numbers 1 through 5,000 to label each of 5,000 sets of 100 coin tosses. In other words, the simulation contains 5,000 trials.
- Column E contains the number of heads in each of the 5,000 trials. Cell E2 counts the number of heads from the 100 coin tosses in column B. To trick Excel into repeating this another 4999 times, I used a *Data Table* from the *What-if-Analysis* under the *Data* menu. I highlighted the area D2:E5001, selected *Data Table*, inserted D1 (or any other cell would do) as the *Column input cell*, and pressed OK.
- Column G contains the numbers 0 through 100 as labels for 0 of 100 heads, 1 of 100 heads, etc.
- Column H counts the number of the 5000 trials that gave the number of heads indicated in column G.
- Column I estimates the fraction of trials that gave the number of heads indicated in column G.

	A	B	C	D	E	F	G	H	I	J
1	100 tosses			5000 simulation trials			0	0	0.000	
2	1	0.962		1	54		1	0	0.000	
3	2	0.123		2	51		2	0	0.000	
4	3	0.733		3	48		3	0	0.000	
5	4	0.951		4	55		4	0	0.000	
6	5	0.782		5	51		5	0	0.000	
7	6	0.690		6	52		6	0	0.000	
8	7	0.750		7	46		7	0	0.000	
9	8	0.733		8	49		8	0	0.000	
10	9	0.869		9	56		9	0	0.000	

	A	B	C	D	E	F	G	H	I
1	100 tosses			5000 simulation trials			0	=COUNTIF(E:E,G1)	=H1/SUM(H:H)
2	1	=RAND()		1	=COUNTIF(B:B,">0.5")		1	=COUNTIF(E:E,G2)	=H2/SUM(H:H)
3	2	=RAND()		2	=TABLE(D1)		2	=COUNTIF(E:E,G3)	=H3/SUM(H:H)
4	3	=RAND()		3	=TABLE(D1)		3	=COUNTIF(E:E,G4)	=H4/SUM(H:H)
5	4	=RAND()		4	=TABLE(D1)		4	=COUNTIF(E:E,G5)	=H5/SUM(H:H)
6	5	=RAND()		5	=TABLE(D1)		5	=COUNTIF(E:E,G6)	=H6/SUM(H:H)
7	6	=RAND()		6	=TABLE(D1)		6	=COUNTIF(E:E,G7)	=H7/SUM(H:H)
8	7	=RAND()		7	=TABLE(D1)		7	=COUNTIF(E:E,G8)	=H8/SUM(H:H)
9	8	=RAND()		8	=TABLE(D1)		8	=COUNTIF(E:E,G9)	=H9/SUM(H:H)
10	9	=RAND()		9	=TABLE(D1)		9	=COUNTIF(E:E,G10)	=H10/SUM(H:H)

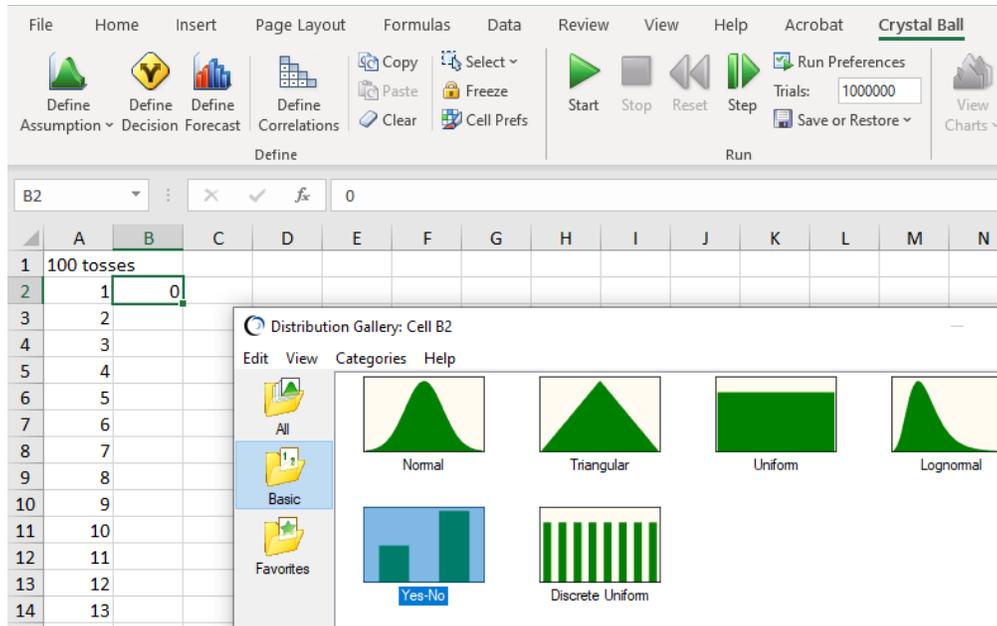
Based on the fractions in column I, I made the distribution below. For comparison, I also included the binomial distribution. The simulated distribution is close to what it should be based on the binomial distribution formula. With more trails, it would get even closer.



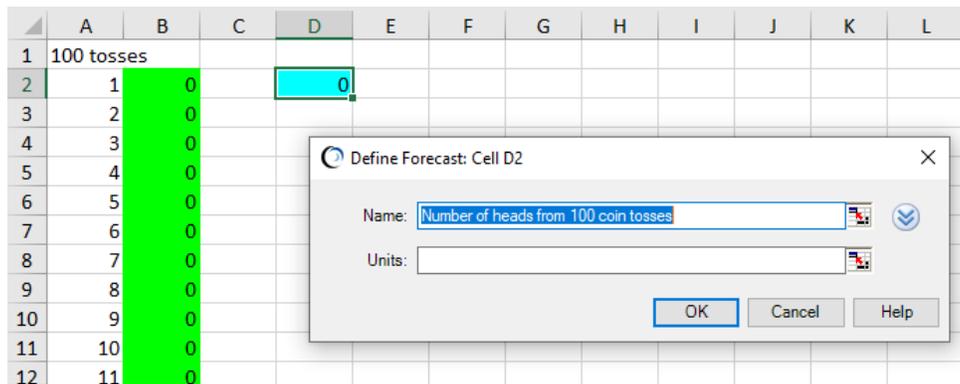
The example shows that it is possible to undertake simple simulations using the imbedded functions in Excel. Excel can also handle other distributions by combining the RAND function with Excel's statistical functions. For example, to draw a random value from a *normal distribution* with an average of X and standard deviation of Y, simply write: =NORM.INV(RAND(),X, Y). But Excel struggles with, e.g., correlations across distributions and a large number of trials. That is when we need more designated simulation tools.

Let us repeat the simulation using Crystal Ball, which is an add-on to Excel that is designed for simulations. Once installed, Crystal Ball gives a separate menu tab in Excel with the simulation tools.

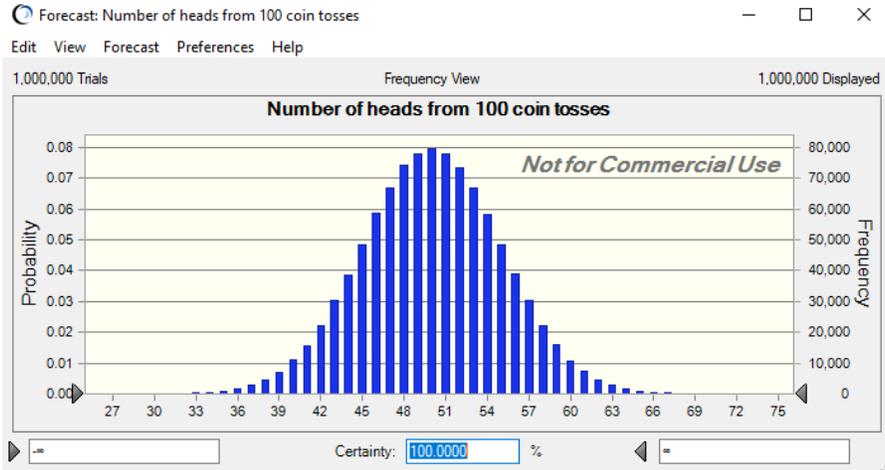
I started with the same column A as earlier, containing the numbers 1 through 100 to label each of the 100 coin tosses. I then clicked on cell B2 and selected *Define Assumption* from the Crystal Ball menu. From the menu of distributions, I chose *Yes-No* and a probability of 50%, and the cell automatically turns lime green. That sets up the simulation of the first coin toss by randomly giving either 0 or 1. Then I copied cell B2 to cells B3 through B101 to set up simulations of another 99 coin tosses. This must be done using the copy and paste functions in the Crystal Ball menu, because the ordinary copy and paste functions in Excel fail to capture the specialized Crystal Ball content. Now, cells B2 through B101 are lime green. We could test the simulation model by pressing *Step* from the Crystal Ball menu, and you would see that all the values in cells B2 through B101 turn randomly either 0 or 1.



Next, I need to count the number of heads among the 100 coin tosses, so I clicked on cell D2 and inserted the formula: =SUM(B:B). I could then simulate 100 coin tosses by pressing *Step*, and cell D2 would update to a value of about 40–60. To record this value for each simulation of 100 coin tosses, I clicked on cell D2, selected *Define Forecast* from the Crystal Ball menu, inserted the name “Number of heads from 100 coin tosses”, and the cell automatically turned turquoise as a reminder that the cell contains an output variable.



For a simulation of one million trials, I inserted 1000000 after *Trials* in the Crystal Ball menu. Then I pressed *Start* to launch one million trials. A box appears with the distribution of the number of heads for each of the one million trials.



I could click on *Extract Data* and select *Trial values* to get the underlying 1,000,000 trial values and make my own histogram to my liking, but this is good enough for now.

You might argue that we did not have to simulate the distribution in this case, because we could simply use the binomial distribution formula. True. But most settings are too complex to fit into a simple framework like the binomial distribution. After a couple of more examples, you will appreciate the flexibility and power of simulations.

2. A stock portfolio

Suppose we have a portfolio of \$30,000 allocated equally across shares in Apple (AAPL), JPMorgan Chase (JPM), and Exxon (XOM), and we want to simulate the value of the portfolio after one year. As a first step, we need to make some assumptions about the following:

- the expected return for each stock,
- the standard deviation of returns for each stock,
- the correlations of returns across the stocks, and
- the shape of the distribution of returns.

I used historical returns to estimate the standard deviations and correlations of returns. Furthermore, I estimated betas, which I plugged into CAPM to estimate expected returns. The snapshot from Excel shows the monthly expected return, monthly standard deviation, and correlations of returns for the three stocks.

The shape of the distribution of returns is not obvious. For short periods, e.g., a day, the returns have approximately a normal distribution, while for long periods, e.g., a year, the return distribution is more skewed, like a lognormal distribution. I chose to simulate the returns one month at a time, and I assumed that the monthly returns follow a *normal distribution*. This approach causes the annual returns to be skewed, as they should be.

As a second step, I constructed the spreadsheet that estimates the value of the portfolio after one year.

- Rows 1–9 contain assumptions.
- Cells B14–D25 allow inputs for investments in the three stocks at the beginning of each month. The first month contains investments of \$10,000 in each stock, while the other months have no further investments. Obviously, I could readily change any of these values.
- Cells E14–G25 contain the monthly returns for each of the stocks. These are the variables that will be simulated.
- Cells H14–J25 contain the end-of-month values for each of the stocks.
- Cells K14–K25 contain the end-of-month values for the portfolio. Cell K25 contains the portfolio value after 12 months and is the output variable of interest.

	A	B	C	D	E	F	G	H	I	J	K
1		AAPL	JPM	XOM							
2	Monthly expected return	0.77%	0.76%	0.68%							
3	Monthly std dev of returns	8.80%	8.70%	8.90%							
4											
5	Correlation matrix										
6		AAPL	JPM	XOM							
7	AAPL	1									
8	JPM	0.44	1								
9	XOM	0.35	0.66	1							
10											
11											
12		Investment beg of month			Return during month			End of month value			
13	Month	AAPL	JPM	XOM	AAPL	JPM	XOM	AAPL	JPM	XOM	Total
14	1	\$10,000	\$10,000	\$10,000	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
15	2	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
16	3	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
17	4	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
18	5	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
19	6	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
20	7	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
21	8	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
22	9	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
23	10	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
24	11	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
25	12	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000

	A	B	C	D	E	F	G	H	I	J	K
1		AAPL	JPM	XOM							
2	Monthly expected return	0.0077	0.0076	0.0068							
3	Monthly std dev of returns	0.088	0.087	0.089							
4											
5	Correlation matrix										
6		AAPL	JPM	XOM							
7	AAPL	1									
8	JPM	0.44	1								
9	XOM	0.35	0.66	1							
10											
11											
12		Investm			Return			End of month value			
13	Month	AAPL	JPM	XOM	AAPL	JPM	XOM	AAPL	JPM	XOM	Total
14	1	10000	10000	10000	0	0	0	=B14*(1+E14)	=C14*(1+F14)	=D14*(1+G14)	=SUM(H14:J14)
15	2	0	0	0	0	0	0	=(H14+B15)*(1+E15)	=(I14+C15)*(1+F15)	=(J14+D15)*(1+G15)	=SUM(H15:J15)
16	3	0	0	0	0	0	0	=(H15+B16)*(1+E16)	=(I15+C16)*(1+F16)	=(J15+D16)*(1+G16)	=SUM(H16:J16)
17	4	0	0	0	0	0	0	=(H16+B17)*(1+E17)	=(I16+C17)*(1+F17)	=(J16+D17)*(1+G17)	=SUM(H17:J17)
18	5	0	0	0	0	0	0	=(H17+B18)*(1+E18)	=(I17+C18)*(1+F18)	=(J17+D18)*(1+G18)	=SUM(H18:J18)
19	6	0	0	0	0	0	0	=(H18+B19)*(1+E19)	=(I18+C19)*(1+F19)	=(J18+D19)*(1+G19)	=SUM(H19:J19)
20	7	0	0	0	0	0	0	=(H19+B20)*(1+E20)	=(I19+C20)*(1+F20)	=(J19+D20)*(1+G20)	=SUM(H20:J20)
21	8	0	0	0	0	0	0	=(H20+B21)*(1+E21)	=(I20+C21)*(1+F21)	=(J20+D21)*(1+G21)	=SUM(H21:J21)
22	9	0	0	0	0	0	0	=(H21+B22)*(1+E22)	=(I21+C22)*(1+F22)	=(J21+D22)*(1+G22)	=SUM(H22:J22)
23	10	0	0	0	0	0	0	=(H22+B23)*(1+E23)	=(I22+C23)*(1+F23)	=(J22+D23)*(1+G23)	=SUM(H23:J23)
24	11	0	0	0	0	0	0	=(H23+B24)*(1+E24)	=(I23+C24)*(1+F24)	=(J23+D24)*(1+G24)	=SUM(H24:J24)
25	12	0	0	0	0	0	0	=(H24+B25)*(1+E25)	=(I24+C25)*(1+F25)	=(J24+D25)*(1+G25)	=SUM(H25:J25)

As the third step, I designated the simulation variables.

- I clicked on cell E14, selected *Define Assumption* from the Crystal Ball menu, and then selected *Normal* from the distribution gallery. I set the mean to =B\$2 and the standard deviation to =B\$3. The cell automatically turned lime green as a reminder that it contains a distribution of values.
- I copied the content of cell E14 to F14 and G14, remembering to use the copy and paste function from the Crystal Ball menu to capture the specialized Crystal Ball content.

- I selected *Define Correlations* from the Crystal Ball menu and inserted the correlations between the variables from the correlation matrix. I could either refer to the cells in the matrix or type in the correlations.
- Then I copied the content of E14–G14 to E15–G25, again remembering to use the copy and paste function from the Crystal Ball menu. I also checked the content of some random cells in E15–G25 by clicking on the cells and *Define Assumptions* to ensure that they contain the desired distributions and correlations.
- Lastly, I clicked on cell K25, selected *Define Forecast* from the Crystal Ball menu, inserted the name “Total value after 12 months”, and the cell automatically turned turquoise.

	A	B	C	D	E	F	G	H	I	J	K	L
1		AAPL	JPM	XOM								
2	Monthly expected return	0.77%	0.76%	0.68%								
3	Monthly std dev of returns	8.80%	8.70%	8.90%								
4												
5	Correlation matrix											
6		AAPL	JPM	XOM								
7	AAPL	1										
8	JPM	0.44	1									
9	XOM	0.35	0.66	1								
10												
11												
12												
13	Month	AAPL	JPM	XOM	Return during							
14	1	\$10,000	\$10,000	\$10,000	AAPL	JPM						
15	2	\$0	\$0	\$0	0%							
16	3	\$0	\$0	\$0	0%							
17	4	\$0	\$0	\$0	0%							
18	5	\$0	\$0	\$0	0%							
19	6	\$0	\$0	\$0	0%							
20	7	\$0	\$0	\$0	0%							
21	8	\$0	\$0	\$0	0%							
22	9	\$0	\$0	\$0	0%							
23	10	\$0	\$0	\$0	0%							
24	11	\$0	\$0	\$0	0%							
25	12	\$0	\$0	\$0	0%							

Define Assumption: Cell E14

Edit View Parameters Preferences Help

Name: AAPL

Normal Distribution

Mean: 1% Std. Dev.: -B\$3

OK Cancel Enter

	A	B	C	D	E	F	G	H	I	J	K
1		AAPL	JPM	XOM							
2	Monthly expected return	0.77%	0.76%	0.68%							
3	Monthly std dev of returns	8.80%	8.70%	8.90%							
4											
5	Correlation matrix										
6		AAPL	JPM	XOM							
7	AAPL	1									
8	JPM	0.44	1								
9	XOM	0.35	0.66	1							
10											
11											
12											
13	Month	AAPL	JPM	XOM	Return during month				End of month value		
14	1	\$10,000	\$10,000	\$10,000	AAPL	JPM	XOM	AAPL	JPM	XOM	Total
15	2	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
16	3	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
17	4	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
18	5	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
19	6	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
20	7	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
21	8	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
22	9	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
23	10	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
24	11	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000
25	12	\$0	\$0	\$0	0%	0%	0%	\$10,000	\$10,000	\$10,000	\$30,000

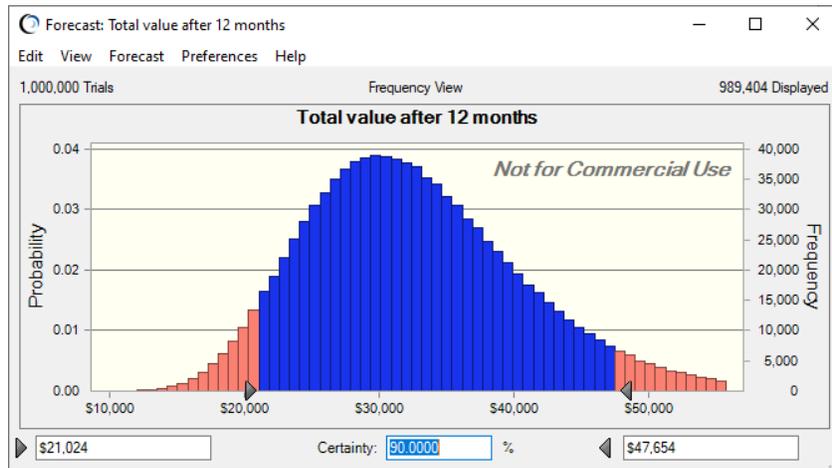
Define Forecast: Cell K25

Name: Total value after 12 months

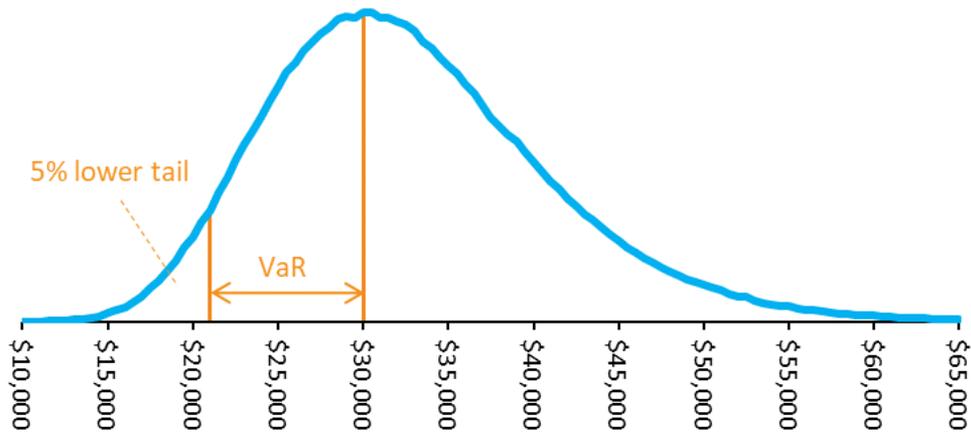
Units:

OK Cancel Help

As the fourth step, I ran the simulation with 1,000,000 trials by pressing *Start* from the Crystal Ball menu. This generates the distribution of the portfolio value after one year.



As a fifth step, I clicked on *Extract Data* and selected *Trial values* to get the 1,000,000 portfolio values after one year. Using these data, I made my own histogram using the standard Excel tools.



You might also ask what the worst-case scenario is over the next year, i.e., how much of the investment is “at risk”? We first need to define “at risk” a little better. Let us agree that it is unlikely that the value will fall in the 5% tail of the distribution – at least that is a commonly used benchmark in the portfolio world. Furthermore, let us set the upper value of the 5% tail as the “minimum” value of the investment at the end of the year. Then the “maximum” that we can lose equals the difference between the current value and the upper value of the 5% tail. We call this the *Value at Risk*, or *VaR* for short, and is depicted in the graph.

3. Projecting corporate cash balances

Firms should periodically project future cash balances to ensure that they are unlikely to run out of cash, which could have severe consequences for their operations.

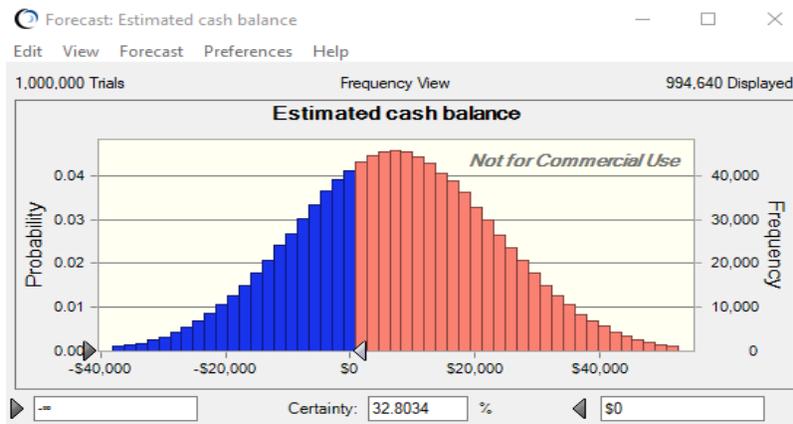
Suppose that a firm has projected the cash balance a year from now with unchanged dividend payments. The snapshots from Excel show that the company is projected to have a cash balance of \$7,283. The positive value might at first seem comforting. However, it represents less than 3% of asset value. Furthermore, the assumptions about sales growth etc. are highly uncertain. Thus, we need to simulate the full distribution of the cash balance.

Using *Define Assumption* from the Crystal Ball menu, the variables in column D have been assigned normal distributions with means given in column E and standard deviations given in column F. Then using *Define Forecast* from the Crystal Ball menu, cell C13 is assigned to be the output variable.

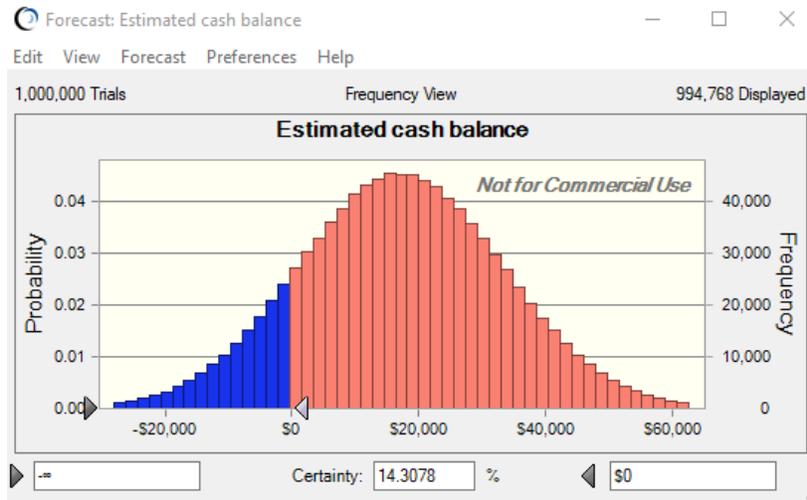
	A	B	C	D	E	F	G
1		2022	2023E		Avg	Std dev	
2							
3	Sales	\$301,535	\$331,689	10%	10%	4%	Growth
4	Variable costs	\$239,919	\$265,351	80%	80%	3%	Fraction of sales
5	Fixed costs	\$32,941	\$32,941	0%	0%	2%	Growth
6	Interest expense	\$5,233	\$5,233				
7	Profit	\$23,442	\$28,164				
8	Tax (21%)	\$4,923	\$5,914				
9	Net profit	\$18,519	\$22,249				
10	Dividend	\$10,000	\$10,000				
11							
12							
13	Cash	\$16,947	\$7,283				Plug
14	Non-cash current assets	\$96,451	\$106,140	32%	32%	3%	Fraction of sales
15	PP&E	\$155,942	\$175,000				Investment plan
16	Total assets	\$269,340	\$288,423				
17							
18	Current liabilities	\$90,995	\$99,507	30%	30%	3%	Fraction of sales
19	LT debt	\$73,104	\$73,000				
20	Equity	\$103,667	\$115,916				
21		\$267,766	\$288,423				

	A	B	C	D	E	F	G
1		2022	2023E		Avg	Std d	
2							
3	Sales	301535	=B3*(1+D3)	0.1	0.1	0.04	Growth
4	Variable costs	239919	=D4*C3	0.8	0.8	0.03	Fraction of sales
5	Fixed costs	32941	=B5*(1+D5)	0	0	0.02	Growth
6	Interest expense	5233	5233				
7	Profit	=B3-B4-B5-B6	=C3-C4-C5-C6				
8	Tax (21%)	=0.21*B7	=0.21*C7				
9	Net profit	=B7-B8	=C7-C8				
10	Dividend	10000	10000				
11							
12							
13	Cash	16947	=C21-C15-C14				Plug
14	Non-cash current assets	96451	=D14*C3	0.32	0.32	0.03	Fraction of sales
15	PP&E	155942	175000				Investment plan
16	Total assets	=B13+B14+B15	=C13+C14+C15				
17							
18	Current liabilities	90995	=D18*C3	0.3	0.3	0.03	Fraction of sales
19	LT debt	73104	73000				
20	Equity	103667	=B20+C9-C10				
21		=B18+B19+B20	=C18+C19+C20				

The simulation produced a full distribution of the cash balance in a year. The distribution shows that the probability of a cash shortfall is 32.8%. This is, obviously, troubling.



Perhaps eliminating the dividend resolves the problem. To check, I set the dividend to zero and reran the simulation. Now the probability of a cash shortfall is 14.3%. Apparently, eliminating the dividends helps substantially.



I also extracted the data and made overlaying distributions. Evidently, the firm needs to take actions even beyond cutting the dividends to ensure liquidity. Perhaps it could slow the growth or raise some additional funds. Either way, it is good that we caught the potential problem now, because it is easier to resolve a liquidity problem before it becomes too severe.

